Algebraic perturbation theory for polar fluids: A model for the dielectric constant

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An alternative microscopic theory of the dielectric constant is proposed based on the algebraic technique due to Ruelle and the statistical mechanical perturbation approach. The resulting analytical expression is third order in the interaction energy and fifth order in the density. A comparison with other theoretical models and computer simulations for dipolar hard spheres and Stockmayer fluids is presented. The theory is equally applicable for describing the initial magnetic susceptibility of ferrofluids, in which case it is in good agreement with experimental data. [S1063-651X(99)01804-8]

I. INTRODUCTION

The theory of the dielectric constant ϵ pioneered in 1912 by Debye [1] still remains an intensively developing area. Its fundamental and practical importance has motivated a lot of theoretical work and computer simulation studies (for reviews see, e.g., [2] and [3]). However, agreement between various models and simulations remains rather poor, especially for condensed systems at relatively low temperatures when interparticle interactions play an increasingly important role. In a seminal paper [1], Debye obtained ϵ by studying the response of a dilute gas of particles to an applied external electrical field $\mathbf{E}_{\text{ext}}$ (in the same fashion as Langevin's work on a paramagnetic gas in an external magnetic field). Subsequent models mainly used an approach which relates ϵ to the properties of the system in the *absence* of $\mathbf{E}_{\text{ext}}$. In these theories each particle is regarded as a source of a field acting on its neighbors and ϵ describes the response of the system to this field.

In the present paper we propose a theory of the dielectric constant for a polar nonpolarizable fluid in the spirit of the original Debye-Langevin approach: we study the response of a dielectric system to a weak external field, taking into account dipole-dipole interactions. These are long ranged and anisotropic and represent the main source of difficulties arising in theoretical descriptions and simulation studies. Our model is based on a statistical mechanical perturbation theory and a powerful "algebraic technique" due to Ruelle [4]. In view of the similarity between electrical and magnetic quantities, this model is straightforwardly applicable for the description of the static initial magnetic susceptibility of ferrofluids.

II. MODEL

We consider a polar fluid, in which polarizability effects are neglected, as a system of N hard spheres with point dipoles at their centers. The fluid is contained in a volume V at temperature T and placed into a weak external homogeneous electric field $\mathbf{E}_{\text{ext}}$. Each particle i is characterized by a five-dimensional vector $\widetilde{\mathbf{r}}_i = (\mathbf{r}_i, \boldsymbol{\omega}_i)$, where $\mathbf{r}_i$ is its radius vector and $\boldsymbol{\omega}_i = (\theta_i, \varphi_i)$ denotes the orientation of its dipole moment $\mathbf{s}_i$. We assume that particles are identical, so that d is the hard-sphere diameter and $|\mathbf{s}_i| = s$ (extention to mixtures is

straightforward). The potential energy for an arbitrary configuration consists of an interparticle interaction energy and an external field contribution, $U(\widetilde{\mathbf{r}}^N) = U_0(\widetilde{\mathbf{r}}^N) + U_1(\boldsymbol{\omega}^N)$,

$$U_0 = \sum_{i<j} [u_{d,ij} + u_{ij}^{(dd)}], \tag{1}$$

$$U_1 = -sE_{\text{ext}} \sum_i \cos\theta_i. \tag{2}$$

Here $u_{d,ij} = u_d(r_{ij})$ is the hard-sphere interaction, r_{ij} the particle separation, θ_i the angle between $\mathbf{s}_i$ and $\mathbf{E}_{\text{ext}}$, and

$$u_{ij}^{(dd)} = \frac{s^2}{r_{ij}^3} D(i,j)$$

the dipole-dipole potential in which the angular part is given by

$$D(i,j) = \hat{\mathbf{s}}_i \hat{\mathbf{s}}_j - 3(\hat{\mathbf{s}}_i \hat{\mathbf{r}}_{ij})(\hat{\mathbf{s}}_j \hat{\mathbf{r}}_{ij}),$$

where $\hat{\mathbf{x}}$ denotes a unit vector $\mathbf{x}$. The formulation of any perturbation theory starts with decomposing the system into a reference and perturbative parts. Since we are aiming at a detailed description of the influence of interparticle interactions on the dielectric constant, it is reasonable to include them in a reference model. The latter is characterized by the energy U_0 and represents the system of dipolar hard spheres in a zero field. The interaction with an external field is treated as a perturbation. Introducing the Mayer function

$$f_i = e^{\alpha \cos\theta_i} - 1,$$

where

$$\alpha \equiv \frac{sE_{\text{ext}}}{k_B T},$$

we write the configuration integral as

$$Q = \int d\widetilde{\mathbf{r}}^N e^{-\beta U_0} \left[1 + \sum_{n=1}^N \left(\sum_{1 \le i_1 < \cdots < i_n \le N} f_{i_1} \cdots f_{i_n} \right) \right],$$

$$\beta = \frac{1}{k_B T}. \qquad (3)$$

The same quantity for the reference model reads

$$Q_0 = \int d\tilde{\mathbf{r}}^N e^{-\beta U_0}.$$

Then Eq. (3) becomes

$$\frac{Q}{Q_0} = 1 + \sum_{n=1}^{N} \left[\int d\tilde{\mathbf{r}}^N \frac{e^{-\beta U_0}}{Q_0} \left(\sum_{1 \le i_1 < \cdots < i_n \le N} f_{i_1} \cdots f_{i_n} \right) \right].$$

Each term of the sum is a thermal average over the reference model of the quantity in the round brackets. This implies that introducing the n-body reference correlation function $g_n^0(\tilde{\mathbf{r}}^n)$, we can rewrite this expression as

$$\frac{Q}{Q_0} = 1 + \sum_{n=1}^{N} \frac{1}{n!} \left(\frac{\rho}{4\pi} \right)^n \int d\tilde{\mathbf{r}}^n \left(\prod_{i=1}^{n} f_i \right) g_n^0(\tilde{\mathbf{r}}^n), \qquad (4)$$

where $\rho = N/V$ is the number density of the fluid. The standard scheme of the perturbation approach (see, e.g., [5]) is based on the expansion of Q in terms of the Mayer functions. However, in our case, this is not appropriate, since f_i is purely orientational and therefore does not compensate for the long-range behavior of $g_n^0 : g_n^0 \to 1$ when mutual separations between particles become large.

Instead of the standard route, we use an approach based on the algebraic technique proposed by Ruelle [4] (see also [6]). Not entering into the way in which this technique is established, we merely formulate here its main result and outline the way it can be applied to our problem. Consider a series

$$a(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!} a_n, \qquad (5)$$

where z is a formal parameter and the coefficients a_n are given by

$$a_0 = 1, \quad a_n = \int d\tilde{\mathbf{r}}^n q_n^a(\tilde{\mathbf{r}}^n), \quad n \ge 1, \qquad (6)$$

$q_n^a(\tilde{\mathbf{r}}^n)$ being arbitrary real functions. It is rigorously proved in [4] that $a(z)$ can be rewritten in the form of an *exponential* of some other series

$$a(z) = \exp[b(z)] = \exp\left[\sum_{n=0}^{\infty} \frac{z^n}{n!} b_n \right], \qquad (7)$$

where

$$b_0 = 0, \quad b_n = \int d\tilde{\mathbf{r}}^n q_n^b(\tilde{\mathbf{r}}^n), \quad n \ge 1, \qquad (8)$$

and the functions q_n^b are unambiguosly constructed algebraic combinations of $q_n^a : q_n^b = \mathcal{R}(q_1^a, \ldots, q_n^a)$. It is worth emphasizing that this method is a rigorous mathematical procedure based on a homomorphism of the algebra of infinite-dimensional vectors

$$\vec{q^a} = (\text{const}, q_1^a(\tilde{\mathbf{r}}_1), q_2^a(\tilde{\mathbf{r}}_1, \tilde{\mathbf{r}}_2,), \ldots, q_n^a(\tilde{\mathbf{r}}_1, \ldots, \tilde{\mathbf{r}}_n), \ldots)$$

into the algebra of power series $a(z)$.

Comparing Eq. (4) with Eqs. (5) and (6) we identify

$$z \equiv \frac{\rho}{4\pi}, \quad a(z) \equiv \frac{Q}{Q_0}, \quad q_n^a \equiv \left(\prod_{i=1}^{n} f_i \right) g_n^0, \quad n = 1, 2, \ldots.$$

Thus,

$$\frac{Q}{Q_0} = \exp\left[\sum_{n=0}^{\infty} \frac{(\rho/4\pi)^n}{n!} b_n \right]. \qquad (9)$$

The first terms of the sequence $\{q_n^b\}$ read

$$q_1^b = f_1 g_1^0 = f_1, \qquad (10)$$

$$q_2^b = \left(\prod_{i=1}^{2} f_i \right) [g_2^0 - 1] \equiv \left(\prod_{i=1}^{2} f_i \right) h_2^0, \qquad (11)$$

$$q_3^b = \equiv \left(\prod_{i=1}^{3} f_i \right) \left[h_3^0 - \sum_{1 \le i < j \le 3} h_2^0(\tilde{\mathbf{r}}_i, \tilde{\mathbf{r}}_j) \right]. \qquad (12)$$

We stress that taking into account *each term* in Eq. (9) is equivalent to taking into account an *infinite* number of terms in Eq. (4). The configurational free energy reads

$$\beta \mathcal{F} = \beta \mathcal{F}_0 - \sum_{n=1}^{\infty} \frac{(\rho/4\pi)^n}{n!} b_n, \qquad (13)$$

where $\mathcal{F}_0$ is the reference free energy. Equation (13) is exact: no appoximations have been imposed so far.

For the derivation of the dielectric constant we need to find the linear response of the system to a weak external field which corresponds to keeping only the leading term in the expansion of the free energy in powers of α. Since $\mathcal{F}$ is an even function of the field, we should cut off this series, keeping the term $O(\alpha^2)$ and neglecting all the rest. The cutoff is only possible if the b_n contain converging integrals. This is really the case since g_n^0 in Eq. (4) are replaced by $h_n^0 = g_n^0 - 1$ in Eqs. (11) and (12), vanishing at large separations, thus ensuring convergence of Eq. (13) for $n \ge 2$. Substituting Eqs. (10)–(12) into Eq. (8) and expanding the Mayer functions to second order in α we obtain, for the free energy,

$$\beta \mathcal{F} = \beta \mathcal{F}_0 - N \frac{\alpha^2}{6} - \frac{1}{2} \left(\frac{\rho}{4\pi} \right)^2 b_2 + o(\alpha^4). \qquad (14)$$

The second term gives the energy of independent dipoles in an external field E_{ext} (cf. paramagnetic Langevin gas) while the third one accounts for interparticle interactions in the presence of E_{ext}. The pair correlation function of dipolar hard spheres reads [5]

$$g_2^0(r_{12}, \boldsymbol{\omega}_1, \boldsymbol{\omega}_2) = g_d(r_{12}) + \sum_{m=1}^{\infty} (\beta s^2)^m g_m(r_{12}, \boldsymbol{\omega}_1, \boldsymbol{\omega}_2), \qquad (15)$$

where

$$g_1 = -\frac{g_d(r_{12})}{r_{12}^3}D(1,2), \tag{16}$$

$$g_2 = \frac{1}{2}\frac{g_d(r_{12})}{r_{12}^6}D^2(1,2) \tag{17}$$

$$+\frac{1}{6}\rho D(1,2)\int d\mathbf{r}_3 \frac{1+3\cos\alpha_1\cos\alpha_2\cos\alpha_3}{(r_{13}r_{23})^3}g_d(123) \tag{18}$$

$$+\frac{1}{3}\rho\Delta(1,2)\int d\mathbf{r}_3 \frac{3\cos^2\alpha_3-1}{(r_{13}r_{23})^3}g_d(123). \tag{19}$$

Here $\Delta(1,2)=\hat{s}_1\hat{s}_2$, $g_d(r)$ and $g_d(123)$ are the two- and three-body hard-sphere correlation functions and α_1, α_2, α_3 are the angles of the triangle formed by the three particles [7]. A cutoff of Eq. (15) at $m=2$ implies

$$b_2 = \alpha^2 V \sum_{m=0}^{2} b_2^{(m)},$$

where

$$b_2^{(0)} = \int d\mathbf{r}_{12}\int d\boldsymbol{\omega}_1 d\boldsymbol{\omega}_2 \cos\theta_1\cos\theta_2[g_d(r_{12})-1],$$

$$b_2^{(1)} = (\beta s^2)\int d\mathbf{r}_{12}\int d\boldsymbol{\omega}_1 d\boldsymbol{\omega}_2 \cos\theta_1\cos\theta_2 g_1,$$

$$b_2^{(2)} = (\beta s^2)^2\int d\mathbf{r}_{12}\int d\boldsymbol{\omega}_1 d\boldsymbol{\omega}_2 \cos\theta_1\cos\theta_2 g_2. \tag{20}$$

Integration over $\boldsymbol{\omega}_1$, $\boldsymbol{\omega}_2$ yields $b_2^{(0)}=0$ and

$$b_2^{(1)} = -\beta s^2\left(\frac{4\pi}{3}\right)^2 \int_{r_{12}>d} d\mathbf{r}_{12}\frac{1}{r_{12}^3}(1-3\cos^2\theta_{12}), \tag{21}$$

where we replaced $g_d(r_{12})$ with the step function

$$\Theta(r_{12}>d)\equiv\Theta_{12}=\begin{cases}1 & \text{for } r_{12}>d,\\ 0 & \text{for } r_{12}<d.\end{cases} \tag{22}$$

We assume that the container is an (infinitely) long cylinder with an axis parallel to the external field. This ensures the absence of a depolarization field inside the sample (the depolarization factor of a long cylinder is zero) and therefore the macroscopic electric field in it is just $\mathbf{E}_{\text{ext}}$. Integration of Eq. (21) over a long cylinder gives

$$b_2^{(1)} = \beta s^2\left(\frac{4\pi}{3}\right)^3.$$

This is a direct manifestation of the long-range nature of dipole-dipole interactions. The last contribution to b_2 given by Eq. (20) contains in turn three terms originating from $g_2(r_{12},\boldsymbol{\omega}_1,\boldsymbol{\omega}_2)$ in Eqs. (17)–(19); in all of those the radial dependence is short ranged ($\sim 1/r^6$) which makes it possible

to replace integration over a cylinder by integration over a sphere. Only the contribution from Eq. (19) survives:

$$b_2^{(2)} = \frac{1}{3}\left(\frac{4\pi}{3}\right)^2 \rho(\beta s^2)^2\gamma_0,$$

where the dimensionless quantity γ_0 is

$$\gamma_0 = \int d\mathbf{r}_{12}\int d\mathbf{r}_3 \frac{3\cos^2\alpha_3-1}{(r_{13}r_{23})^3}g_d(123). \tag{23}$$

For $g_d(123)$ Kirkwood's superposition approximation gives

$$g_d(123)\approx g_d(12)g_d(13)g_d(23)\approx\prod_{1\leq i<j\leq 3}\Theta_{ij}.$$

Equation (23) can then be integrated analytically to give

$$\gamma_0 = \frac{5}{3}\pi^2. \tag{24}$$

Summarizing, we can write the free energy (14) in a closed form,

$$\beta\mathcal{F} = \beta\mathcal{F}_0 - N\frac{\alpha^2}{6} - \frac{\alpha^2}{54}\rho^2 V d^3\left(4\pi\lambda + \frac{5}{3}\pi^2\rho d^3\lambda^2\right), \tag{25}$$

where

$$\lambda = \beta s^2/d^3$$

is a coupling constant characterizing the strength of the dipole-dipole interaction. The macroscopic polarization $\mathbf{P}$ is related to the free energy via [8]

$$\mathbf{P} = -\frac{1}{V}\frac{\partial\mathcal{F}}{\partial\mathbf{E}_{\text{ext}}}\Big|_{N,V,T},$$

yielding

$$P = \frac{\alpha}{3}\rho s\left[1 + \frac{4\pi}{9}\rho d^3\lambda + \frac{5}{27}\pi^2(\rho d^3\lambda)^2\right].$$

Finally, the dielectric constant is obtained from the thermodynamic relationship

$$(\epsilon-1)\mathbf{E}_{ext} = 4\pi\mathbf{P},$$

which results in

$$\epsilon-1 = 3y\left(1 + y + \frac{15}{16}y^2\right), \tag{26}$$

where

$$y \equiv \frac{4\pi}{9}\beta\rho s^2.$$

As in other theoretical models, ϵ in Eq. (26) is a function of only one parameter—y. Physically, one can expect, how-

ever, a dependence on two parameters, λ and the volume fraction $\phi = (\pi/6)\rho d^3$. The ϕ dependence can be introduced if we go beyond the van der Waals step-function approximation for $g_d(r)$ in the perturbative terms of the free energy and set

$$g_d(r) = \Theta(r > d) + \phi G_1(r) + \phi^2 G_2(r) + \cdots,$$

where the functions $G_i(r)$ vanish at r exceeding several particle diameters. $G_1(r)$ can be found by linearizing in density the Ornstein-Zernike equation for hard spheres in the Percus-Yevick approximation

$$h_d(r_{12};\rho) = \begin{cases} \rho \int d\mathbf{r}_3 c(r_{13};\rho=0)h(r_{23};\rho=0) + o(\rho^2) & \text{for } r_{12} > d, \\ -1 & \text{for } r_{12} < d. \end{cases} \tag{27}$$

Here $c_d(r;\rho)$ is a direct pair correlation function and $h_d(r;\rho) \equiv g_d(r;\rho) - 1$. Routine integration gives

$$G_1(r) \equiv \Phi(r) = 8 - 6\left(\frac{r}{d}\right) + \frac{1}{2}\left(\frac{r}{d}\right)^3, \quad d < r < 2d.$$

For $r > 2d$ $\Phi(r) = 0$. The second-order term can be estimated using the ''kinetic theoretical considerations'' of [9], yielding

$$G_2(r) = \frac{1}{2}\Phi^2(r).$$

Thus, both G_1 and G_2 are nonzero at $d < r < 2d$ and zero outside this interval. The only contribution to b_2 affected by this correction is γ_0 given by Eq. (23). The product of step functions in the Kirkwood approximation for $g_d(123)$ is replaced by

$$\prod_{1 \le i < j \le 3} \Theta_{ij} + \phi W_1 + \phi^2 W_2 + \cdots,$$

where W_1 is a sum of products $\Phi_{ij}H_{ij}\Theta_{ik}\Theta_{lm}$ with

$$\Phi_{ij} \equiv \Phi(r_{ij}), \quad H_{ij} \equiv \Theta(r_{ij} > d)\Theta(r_{ij} < 2d)$$

and W_2 contains terms of the type $\Phi_{ij}H_{ij}\Phi_{kl}H_{kl}\Theta_{mn}$. Hence, γ_0 is replaced by

$$\gamma = \gamma_0(1 + a_1\phi + a_2\phi^2).$$

All the integrations can be performed analytically. Both coefficients prove to be positive and small:

$$a_1 \approx 0.036, \quad a_2 \approx 0.372,$$

implying that ϵ is slightly sensitive to the density dependence of γ; the correction to γ_0 does not exceed 2–5%. Equation (26) now becomes

$$\epsilon - 1 = 3y\left[1 + y + \frac{15}{16}(1 + a_1\phi + a_2\phi^2)y^2\right], \tag{28}$$

which is third order in the coupling constant λ and fifth order in the density.

III. RESULTS AND DISCUSSION

Figure 1 shows the dielectric constant as a function of λ for $\rho^* \equiv \rho d^3 = 0.8$ predicted by various theoretical models—Debye [1], Onsager [10], mean-spherical approximation (MSA) [11], linearized hypernetted-chain approximation (LHNC) [12], and the present, algebraic perturbation theory (APT)—and that found in simulation studies [13]. The Debye theory

$$\epsilon - 1 = \frac{3y}{1-y} \tag{29}$$

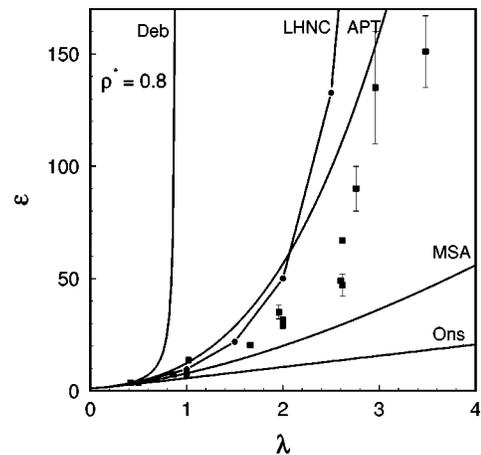has a singular behavior at $y = 1$ which is known to be incorrect [5]. In the Onsager theory

FIG. 1. Dielectric constant as a function of λ for $\rho d^3 \equiv \rho^* = 0.8$. Labels correspond to various theoretical models: Deb, Debye theory; Ons, Onsager theory; MSA, mean-spherical approximation; LHNC, linearized hypernetted-chain approximation; APT, algebraic perturbation theory, Eq. (28). Squares: simulation results [13] for dipolar hard spheres and Stockmayer fluid.

$$\epsilon - 1 = \frac{3}{4}(3y - 1 + \sqrt{1 + 2y + 9y^2}), \qquad (30)$$

the singularity is avoided but ϵ is underestimated. In the MSA ϵ is written in a parametric form

$$\epsilon - 1 = \frac{q(2\xi) - q(-\xi)}{q(-\xi)}, \qquad (31)$$

where ξ is a real root of the equation

$$q(2\xi) - q(-\xi) = 3y, \qquad (32)$$

with the function $q(x)$ given by

$$q(x) \equiv \frac{(1 + 2x)^2}{(1 - x)^4}.$$

In all likelihood MSA also underestimates ϵ [2]. If in the APT orientational correlation is completely ignored, then the reference pair correlation function reduces to that of hard spheres, $g_2^0 = g_d$, providing that $b_2 = 0$. Thus, the APT expression (28) becomes $\epsilon - 1 = 3y$, which is the dielectric constant of the Langevin gas. Exactly the same result follows from all the other above mentioned theories (29)–(31) in the limit of small y.

In view of the long-range nature of dipolar forces, computer simulation of ϵ proved to be a very difficult problem [2,3]. None of the simulation methods gives ϵ for truly infinite systems described by approximate theories. Nevertheless, simulation results can give an idea about the accuracy of various models. Simulations of dipolar hard spheres appear to be technically more difficult than the simulations of a Stockmayer fluid [3], for which a larger amount of data is available. In Fig. 1 simulation results for both systems are shown. It is found in [12] that for $\lambda < 2$, ϵ of a Stockmayer fluid is close to that of equivalent dipolar hard spheres; for larger λ the Stockmayer ϵ is considerably *lower* than that of the corresponding hard-sphere system.

Real molecules usually have both dipole and quadrupole moments which makes a straightforward comparison of APT with real dielectric liquids problematic. However, by changing from the electric to magnetic language, APT can be compared with experimental data on the initial susceptibility χ of ferrofluids in which quadrupole interactions are absent. Thus, **s** is now a magnetic moment of a ferroparticle, $\epsilon - 1$ becomes $\mu - 1 = 4\pi\chi$, and $y = (4\pi/3)\chi_L$, where

$$\chi_L(T) = \frac{1}{3}\frac{\rho s^2}{k_B T} \qquad (33)$$

is the Langevin susceptibility. Figure 2 shows the temperature dependence of the initial susceptibility for a colloidal solution of magnetite in kerosene predicted by various models and the recent experimental results of [14]. To obtain
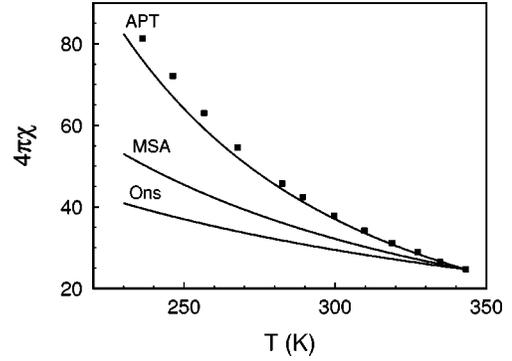

FIG. 2. Temperature dependence of the initial magnetic susceptibility of a ferrofluid. Squares, experiment [14]; solid lines, theoretical predictions (notations are the same as in Fig. 1). Comparison with experiment is made by adjusting the corresponding theoretical χ to the experimental value $4\pi\chi^{\text{ref}} = 24.7$ at the temperature $T^{\text{ref}} = 343.15$ K.

$\chi_L(T)$ we assume that at the highest examined temperature $T^{\text{ref}} = 343.15$ K, where interparticle interactions are at minimum, the experimental value $4\pi\chi^{\text{ref}} = 24.7$ is described by Eq. (26). Solving this cubic equation for y we find $4\pi\chi_L^{\text{ref}} \approx 4.86$. Then, for other temperatures using Eq. (33) we have

$$\chi_L(T) = \frac{T^{\text{ref}}}{T}\chi_L^{\text{ref}}[1 - \beta_1(T - T^{\text{ref}})], \qquad (34)$$

where the term in square brackets takes into account the thermal expansion of kerosene (with the expansion coefficient $\beta_1 \approx 0.9 \times 10^{-3}$ K^{-1}). Figure 2 shows a good agreement of APT with experimental data for the whole temperature range studied. In the same figure predictions of the Onsager theory and the MSA are also shown. To be consistent we use the same procedure for each of these models, adjusting the corresponding theoretical $4\pi\chi$ to the experimental value $4\pi\chi^{\text{ref}} = 24.7$ at $T^{\text{ref}} = 343.15$ K to calculate χ_L^{ref}. This implies that the reference Langevin susceptibility χ_L^{ref} calculated for different models will be different. For other temperatures $\chi_L(T)$ is found from Eq. (34). It is seen that the agreement with the Onsager theory and the MSA is poor.

In conclusion, we have proposed a microscopic model for the dielectric constant of a polar nonpolarizable fluid. The resulting analytical expression is third order in the interaction energy and fifth order in the density. The model proves to be in fair agreement with computer simulation data on the dipolar hard spheres and the Stockmeyer fluids. The model has been also applied to ferrofluids: theoretical predictions of the initial magnetic susceptibility are shown to be in good agreement with experiment.

[1] P. Debye, Phys. Z. **13**, 97 (1912).

[2] G. Stell, G. N. Patey, and J. S. Hoye, Adv. Chem. Phys. **48**, 183 (1981).

[3] S. W. de Leeuw, J. W. Perram, and E. R. Smith, Annu. Rev. Phys. Chem. **37**, 245 (1986).

[4] D. Ruelle, *Statistical Mechanics: Rigorous Results* (Benjamin, New York, 1969).

[5] J. A. Barker and D. Henderson, Rev. Mod. Phys. **48**, 587 (1976).

[6] B. V. Zelener, G. E. Norman, and V. S. Filinov, *Perturbation Theory and Pseudopotential in Statistical Thermodynamics* (Nauka, Moscow, 1981).

[7] There is a misprint in Eq. (7.183) of [5]: $(r_{13}r_{23})$ there should be raised to the power 3, not 2. I thank Professor D. Henderson for confirming this correction.

[8] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continu-ous Media* (Pergamon, Oxford, 1984).

[9] S. Shinomoto, J. Stat. Phys. **32**, 105 (1983).

[10] L. J. Onsager, J. Am. Chem. Soc. **58**, 1486 (1936).

[11] M. S. Wertheim, J. Chem. Phys. **55**, 4291 (1971).

[12] G. N. Patey, D. Levesque, and J. J. Weis, Mol. Phys. **38**, 219 (1979).

[13] E. L. Pollock and B. J. Alder, Physica A **102**, 1 (1980); D. J. Adams, Mol. Phys. **40**, 1261 (1980); D. J. Adams and E. M. Adams, *ibid.* **42**, 907 (1981); M. Neumann, *ibid.* **50**, 841 (1983); G. N. Patey, D. Levesque, and J. J. Weis, *ibid.* **45**, 733 (1982); S. W. de Leeuw, B. Smit, and C. P. Williams, J. Chem. Phys. **93**, 2704 (1990); P. G. Kusalik, Mol. Phys. **76**, 337 (1992); P. G. Kusalik, M. E. Mandy, and I. M. Svishchev, J. Chem. Phys. **100**, 7654 (1994).

[14] A. F. Pshenichnikov and A. V. Lebedev, Colloid J. USSR **57**, 800 (1995).